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INTRODUCTION

OBJECTIVES

Symbolic Regression (SR) is an established Machine Learning technique for identifying optimal mathematical expressions that can describe relationships within a given data structure. However, SR is rarely used in health economic modelling. Several SR algorithms are available from different software.

This research aims at comparing the performance of an artificial intelligence (AI) determined mapping algorithm with traditional regression approaches using data from the EQ-5D-3L and the cancer (non-small cell lung cancer (NSCLC)) specific EORTC-QLQ-C30 (QLQ-C30). This research also compares several different SR approaches with traditional regression estimation.

METHODS

Data from 3 Randomized Controlled Trials (RCTs) in NSCLC reported previously developed mapping models between the QLQ-C30 and the EQ-5D-3L (Jang 2010, Crott 2018, Khan & Morris 2014). The same data were used in SR analyses using four types of software: TuringBot, Mathematica® (through Data Modeler®), GPLEarn and PySR (modules in Python). For comparing performance with the standard regression model, the root mean square error (RMSE), 1-R² and mean absolute error (MAE) and the complexity score were used. The model used EQ-5D utilities as the response regressed against 15 QLQ-C30 domain scores.

RESULTS

Table 1: QLQ-C30 Quality of Life Profiles (Mean Scores)

QLQ-C30 Function	Topical N=2379	Soccar N=994	Jang N=172
Physical (PF)	54.15	76.62	73.37
Relational (RF)	48.49	68.52	67.44
Emotional (EF)	73.63	77.43	75.48
Cognitive (CF)	77.02	82.8	80.03
Social (SF)	65.82	73.75	73.64
Overall QoL (QL)	51.19	65.11	65.89
(FA) fatigue	49.74	34.65	40.82
(NV) nausea/vomiting	10.71	12.09	7.56
(PA) pain	26.50	21.36	25.67
(DY) dyspnoea	51.50	34.44	31.19
(SL) sleep	29.84	25.01	34.87
(AP) appetite	36.89	22.33	22.66
(CO) constipation	20.69	18.64	18.48
(DI) diarrhoea	15.10	6.20	13.36
(FI) financial problems	10.84	20.79	23.05

Table 2: Model Performance across Software and Regression Models

		Topical	Soccar	Jang
Benchmark OLS	R ²	0.640	0.630	0.584
	MAE	0.132	0.100	0.141
	RMSE	0.179	0.141	0.199
Symbolic Regression	R ²	0.619	0.585	0.642
	MAE	0.137	0.104	0.135
	RMSE	0.184	0.148	0.169
TuringBOT	R ²	0.389	0.312	0.331
	MAE	0.174	0.139	0.178
	RMSE	0.233	0.191	0.232
Symbolic Regression	R ²	0.348	0**	0.199
	MAE	0.181	0.167	0.189
	RMSE	0.242	0.231	0.267
Symbolic Regression	R ²	0.656	0.670	0.735
	MAE	0.128	0.092	0.109
	RMSE	0.175	0.132	0.148
Published Best values	R ²	0.75	0.71	NA
	MAE	0.10	0.13	0.104
	RMSE	0.09	0.11	0.073
Regression Model		Beta-binomial	Beta-binomial	Piecewise OLS

* Best accuracy results in bold (through DataModeler®).
** regression yielded only a constant mean utility but no equation.
Note: Higher R² is considered better, lower RMSE and MAE are considered better.

AI determined methods through SR using the Mathematica® algorithms demonstrated better performance compared to the standard linear regression methods. SR provided higher R² – in some cases (Jang), the R² was about 26% higher. Consequently, mean predicted utilities were closer to the observed mean utilities (as were the standard deviations). Compared to OLS, Mathematica® was able to improve the accuracy of the prediction while other SR algorithms were less able to reach that goal. It also resulted in the same mean and the closest standard deviation of the estimated utilities compared to the observed utilities.

Table 3: OLS and SR Mean Predicted Utilities

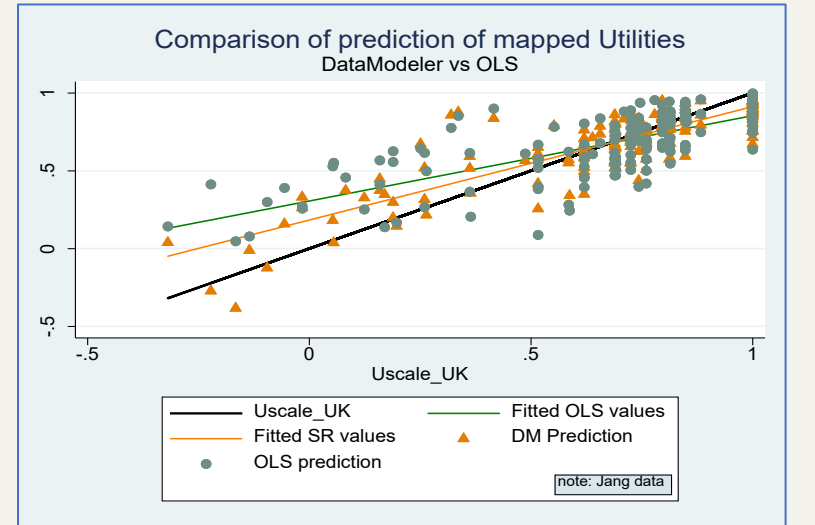
	Topical Mean (SD)	SOCCAR Mean (SD)	Jang Mean (SD)
Observed utilities	0.607 (0.297)	0.750 (0.230)	0.676 (0.284)
Benchmark OLS prediction	0.607 (0.238)	0.750 (0.182)	0.676 (0.210)
SR prediction TuringBot	0.605 (0.248)	0.750 (0.170)	0.671 (0.239)
SR prediction GPLEarn	0.618 (0.214)	0.741 (0.175)	0.706 (0.153)
SR prediction PySR	0.611 (0.148)	0.750 (0) constant	0.670 (0.137)
SR prediction DataModeler	0.607 (0.241) *	0.750 (0.189)*	0.677 (0.243)*

Note: * closest symbolic regression results to observed mean and standard deviation.

Best SR Equation (Jang Data)

$$\begin{aligned} & -0.12 + (2.44 \times 10^{-3}) eFscore - (2.45 \times 10^{-3}) pAScore + (8.57 \times 10^{-2}) \sqrt{pFscore} - (1.41 \times 10^{-3}) pFscore + \\ & \frac{(2.85 \times 10^{-3}) pAScore}{6} - fAScore + nVscore + pFscore - 4 \cdot pAScore + pFscore + \frac{(3.27 \times 10^{-3}) (135.33 \cdot e^3 \sqrt{pFscore} + 3 pFscore)}{8 - cFscore} \\ & \frac{(2.53 \times 10^{-3}) pAScore}{-6 \cdot nVscore + pFscore - fFscore} + (1.61 \times 10^{-3}) rFscore + \frac{(2.14 \times 10^{-3}) \left(9.13 - \frac{eFscore}{6} + \frac{eFscore^2}{216} + 2 fFscore \right)}{6 - 4 cFscore - nVscore} \end{aligned}$$

Figure 1: Comparison of Goodness of Fit of Best SR versus OLS in the Jang Data Sample



CONCLUSIONS

Symbolic Regression may outperform standard linear regression. It depends very much on the various SR algorithms. We found that in this case the genetic algorithm in Mathematica provided the best results. Mean predicted utilities were close to observed in most cases. SR could also be helpful in identifying the set of influential explanatory variables. More recent alternative SR methods like Bayesian, neural networks, or transformer-based approaches could improve on the current results. Further research in other disease areas and other generic quality of life measures is warranted as with more advanced regression methods like mixture models or spline regression is warranted.

References

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